Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Level 1 Mathematics and Statistics RAS 2023

91947 Demonstrate mathematical reasoning

EXEMPLAR

Excellence

TOTAL 24

QUESTION ONE

(a) Find the value of T in the formula $T = \pi\sqrt{\dfrac{h\sin x}{g}}$ when $h = 2.5$, $g = 9.81$, $x = 75°$,

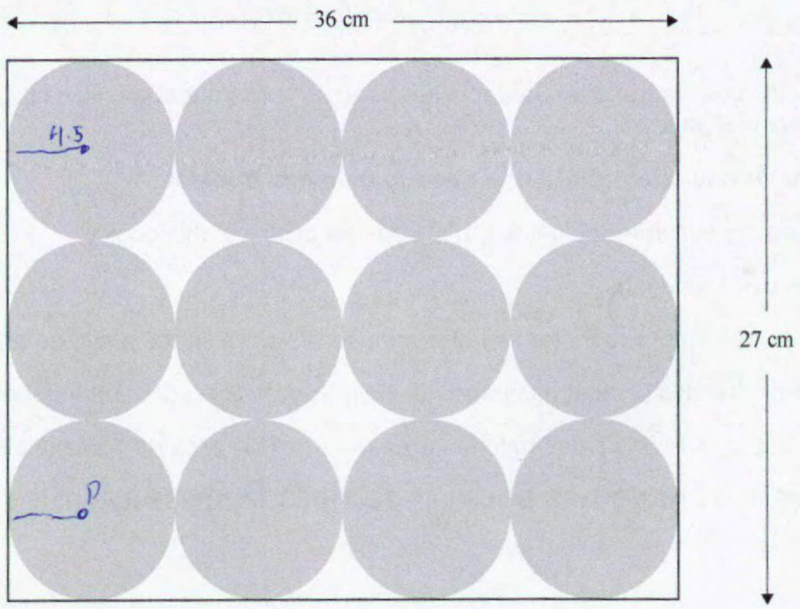
giving your answer correct to **four decimal places**.

$T = \pi\sqrt{\dfrac{2.5\sin(75)}{9.81}} = 1.5587.$ (4dp)
(use calculator)

(b) The diagram below shows the top view of a rectangular box containing 12 cylindrical tins.
The tins are all just touching each other and the sides of the box. Each tin is 15 cm high.
Each tin has a label going all the way around its side, but not on the top or bottom.
The box has dimensions of 27 cm by 36 cm by 15 cm.

Diagram is NOT to scale

$\pi r^2 h$ = volume
$2\pi rh + 2\pi r^2$ = S.A



Source: https://www.thewarehouse.co.nz/p/watties-condensed-tomato-soup-420g/R930548.html

(i) Find the **total area** of the labels of all of the tins in the box.

SA cylinder $= 2\pi rh + 2\pi r^2$.

Since no top or bottom $= -2\pi r^2$. $= 2\pi rh$.

radius $= 36 \div 4 = 9 \div 2 = 4.5 cm^2$. height $= 15$. so $2\pi(4.5)(15) = 135\pi$.

$135\pi \times 12 = 1620\pi$.

$1620\pi = 5089.38 cm^2$. (2dp)

(ii) A different size rectangular box to part (i) has height 15 cm.

The box will also contain 12 cylindrical tins, which are all just touching each other and the sides of the box. The layout of the 12 tins within this box will be the same as in part (i).

Each tin is 15 cm high, and with radius p cm.

Show that the **proportion** of the volume in the box that is NOT occupied by the tins is $\frac{(4-\pi)}{4}$.

Total length $= 8p$, width $= 6p$. volume box $= h \times w \times l$. $l = 15$.

$8p \times 6p \times 15 = 720p^2$.

volume cylinder $= \pi r^2 h$. $r = p$. so $\pi(p)^2(15) = 15p^2\pi \cdot \times 12 = 180p^2\pi$.

$\frac{(720p^2 - 180p^2\pi)}{720p^2 \cancel{180p^2\pi}}$ so factor out p^2.

$\frac{p^2(720 - 180\pi)}{p^2(720)}$

so $\frac{720 - 180\pi}{720}$ ~~÷ 180 on both sides~~

so factor out 180: $\frac{180(4-\pi)}{180 \times 4}$

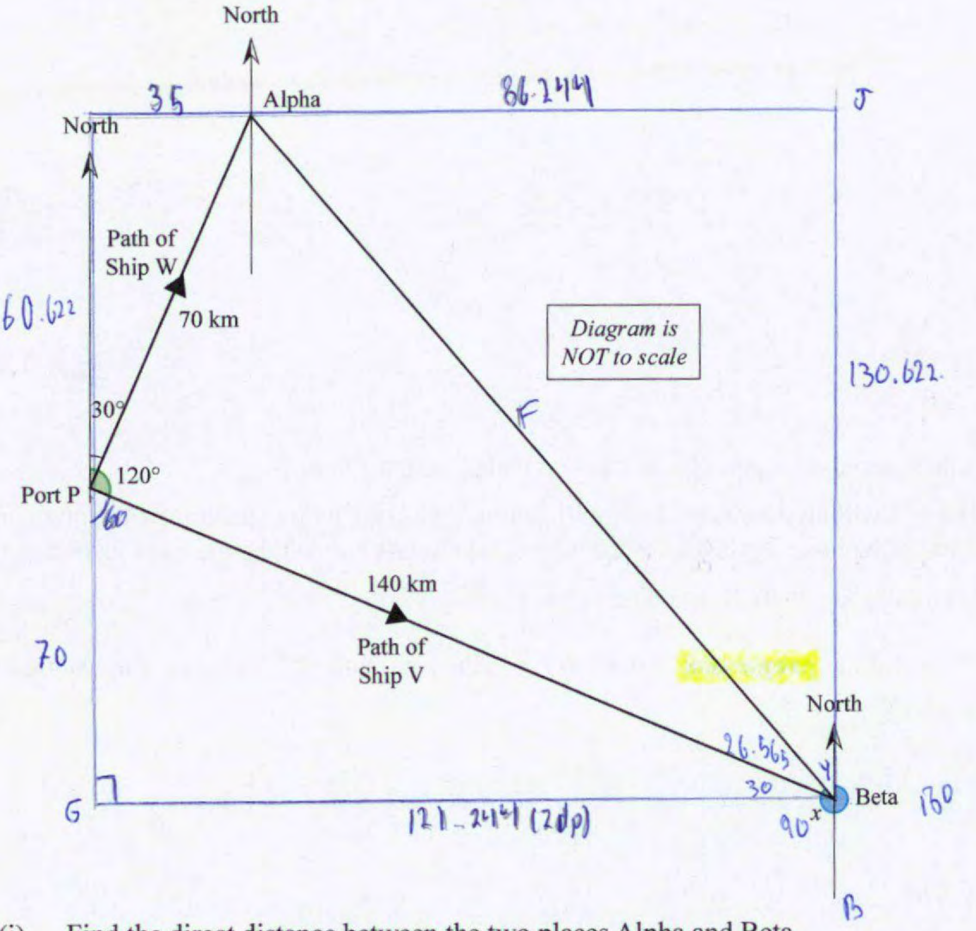
$= \frac{4-\pi}{4}$

Tan e7.

(c) Two ships leave Port P at the same time.

Ship W sails 70 km on a bearing of 030° to reach point Alpha.

Ship V sails 140 km on a bearing of 120° to reach point Beta.



(i) Find the direct distance between the two places Alpha and Beta.

From North to Alpha, use $70\sin(30) = 35$. ∠'s on st line $= 180°$. ∠GPX $= 60°$

From G to Beta use $140\sin(60) = 121.244$ (3dp)

length AlphaJ $= 121.244 - 35 = 86.244$.

for North port p, use ~~70 tan~~ $70\cos(30) = 60.622$ (3dp)

For Port p, G use $140\cos(60) = 70$. $70 + 60.622 = 130.622$

Use $a^2+b^2=c^2$. $\sqrt{130.622^2 + 86.244^2} = c$, $c = 156.525$ km^2.

So from Alpha to beta is 156.525 km^2. (3dp)

or simplier way: $\sqrt{70^2+140^2} = 156.525$ km^2 (3dp)

due to right ∠ triangle

(ii) Find the bearing of Alpha from Beta, shown as angle x in the diagram opposite.

Show your working clearly.

~~i's on st line=180°~~ So North B=180°. ~~∠GxB=90°. Tan⁻¹~~

~~$Tan^{-1}\left(\frac{70}{121.244}\right)=30° \therefore ∠Gxp.$~~

Use Tan⁻¹ ($Tan(y)=\frac{86.244}{130.622}=$ $Tan^{-1}\frac{86.244}{130.622}=y=33.44°$

∠ around point = 360°. 360 - ans = 326.56 so $\therefore$ $\alpha = 326.56°$ (2dp)

(iii) The speed of ship W is k km/hour, where k is a positive constant.

The total time taken for the ships to complete their journeys to Alpha and Beta was four hours.

Find the speed of ship V, giving your answer in terms of k.

Average speed = $\frac{\text{Total distance}}{\text{Total time}}$.

Ship V went from port p to beta at 140km.

Ship W went from port p to Alpha at 70km hour and at k km/h.

So Ship W time = $\frac{\text{distance}}{\text{speed}}$ $\frac{70}{k}$ is time it took so $4-\frac{70}{k}$ is time that ship v can use.

$4-\frac{70}{k}$ = Time available. distance = 140. speed = $\frac{\text{distance}}{\text{time}}$ =

$\frac{140}{4-\frac{70}{k}}$ = speed.

$\frac{4}{1}-\frac{70}{k}=\frac{4k-70}{k}$

So $\therefore$ speed of ship V = $\frac{140}{4-\frac{70}{k}}$, simplify it: $\frac{140}{1} \div \frac{4k-70}{k}$ =

$\frac{140}{1} \times \frac{k}{4k-70} = \boxed{\frac{140k}{4k-70}}$

So $\therefore$ speed of ship V = $\frac{140k}{4k-70}$ (simplified) km/h

$V=\frac{140}{4-\frac{70}{k}}$ (unsimplified) km/h } equivilent

QUESTION TWO

(a) The diagram below shows the top of a table which is in the shape of a regular octagon.

Length AZ = 120 cm. Point Z is at the centre of the octagon.

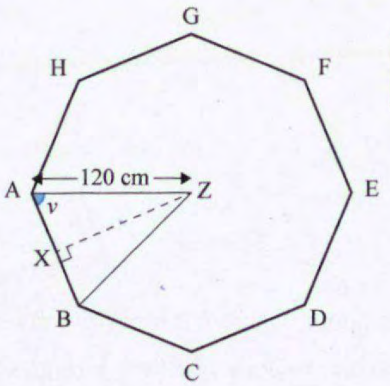


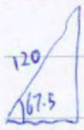
Diagram is *NOT to scale*

(i) Show that the size of v, angle ZAB, is 67.5°.

Show your working clearly.

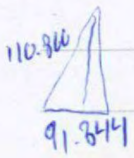
∠'s in octogon sum: $(8-2)180 = 1080 \div 8 = 135$. Bisects $= 135 \div 2 = 67.5°$.
(1dp)

(ii) Find the area of the octagon.



so $120\sin(67.5) = 110.866$ (3dp). = height (cm²)

$\frac{1}{2}$ base = $120\cos(67.5) = 45.922$ cm² so full base = 91.844.



$A = \frac{1}{2} b \times h$

so $\frac{110.866 \times 91.844}{2} = 5091.188$ cm². ×8 tri =

40,729.508 cm² total area. (3dp)

May be 40,729.504 due to rounding but it's 100% 40,729.50. (2dp)

(iii) Another table, made in the same style, has its top in the shape of an n-sided regular polygon.

The length AZ = p cm, where Z is at the centre of the table and A is one of the corners of the table.

Find the area of this new table top, giving your answer in terms of n and p.

So find angle sum: $\frac{(n-2)180}{n}$: angle so

so use

$P\sin\left(\frac{(n-2)180}{n}\right)$ = length,

$P\cos\left(\frac{(n-2)180}{n}\right)$ = base. base x 2 = full base so $2P\cos\left(\frac{(n-2)180}{n}\right)$. (due to bisected it)

Area = $\frac{1}{2}$ bxh.

$\frac{\left(P\sin\left(\frac{(n-2)180}{n}\right)\right)\left(2P\cos\left(\frac{(n-2)180}{n}\right)\right)}{2}$ = one triangle area.

so n sided polygon = multiply by n

so

~~total area = $n\left(P\sin(n-2)180\right.$~~

inside = $(n-2)(180) = \frac{180n-360}{n}$. so

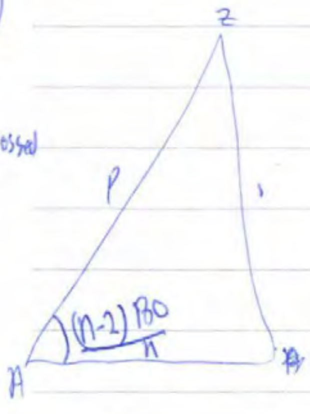
$n\left(\frac{\left(P\sin\left(\frac{180n-360}{n}\right)\right)\left(2P\cos\left(\frac{180n-360}{n}\right)\right)}{2}\right)$

which is same as

$n\left(\frac{\left(P\sin\left(\frac{180n-360}{n}\right)\right)\left(2P\cos\left(\frac{180n-360}{n}\right)\right)}{2}\right)$

equivalent

Note: The n doesn't matter position whether it's $n\left(\frac{\text{working}}{2}\right)$ or $\frac{n(\text{working})}{2}$ which is crossed out part.

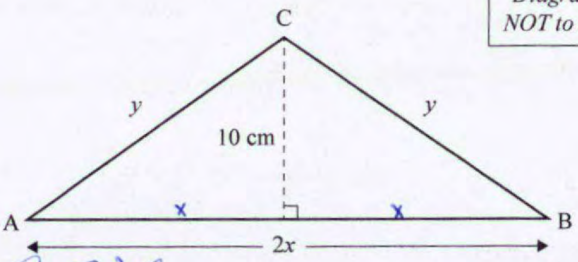


(b) An isosceles triangle ABC has AB = $2x$ cm and AC = BC = y cm.

The perimeter of the triangle ABC is 100 cm.

The length of the perpendicular from C to the line AB is 10 cm.

Diagram is NOT to scale



~~Working at Back~~

(i) Find the length, y, from A to C.

Give your answer in terms of x.

$2x+2y=100$

$2y=100-2x$ or using pythageros, $x^2+100=y^2$. $y=\sqrt{x^2+100}$

$y=\frac{100-2x}{2}=50-x$ using normal algebra, one Pythageros.

both valid. one simplified and other isn't.

(ii) Using Pythagoras' theorem, find the area of the triangle ABC.

Support your answer with full mathematical working.

Working at back, mini Simplified working at bottom of the page in case unavailable. Area = 240 cm²

Please check back working there.

Please note it is in final page, not the first ones of Extra space if required.

$x^2+y^2=10^2$. $\cancel{\frac{2x\times 10}{2}}$ ~~area one~~

While perimeter = $2x+2y=100$, $y=\frac{100-2x}{2}$

So

$x^2+\left(\frac{100-2x}{2}\right)^2=10^2$, $\frac{(100-2x)}{2}\frac{(100-2x)}{2}$ $\frac{10000-400x+4x^2}{4}$

$x^2+\frac{4x^2-400x+10{,}000}{4}=100$

$\frac{4x^2+4x^2-400x+10{,}000}{4}=100$

$8x^2-400x+10{,}000=400$

$8x^2-400x+9600=0$ which is same as

$x^2-50x+1200=0$

I got $x=24$ at end, last page.

So area = $\frac{(10)(2(24))}{2}=240\text{cm}^2$

$a^2+b^2=c^2$, $a=x$, $b=10$, $c=y$. $x^2+10^2=y^2$.

While perimeter: $2x+2y=100$, $2y=100-2x$. $y=50-x$.

$x^2+10^2=y^2$. $x^2+100=(50-x)^2$. $x^2+100=x^2-100x+2500$

$100x=2400$, $x=24$ so $\therefore \frac{1}{2}b\times h$.

$b=2x$, $h=10$. $\frac{2(24)\times 10}{2}=\frac{480}{2}=240\text{cm}^2$

240 cm².

QUESTION THREE

(a) (i) The table below represents points on a particular graph, G_1.

Find the equation of this graph.

x	y
1	20
2	25
3	30
4	35
5	40

)5)5)5)5

1st diff constant so $y - y_1 = m(x - x_1)$ while $m = 5$. $(1, 20)$ $x_1\ y_1$

Linear $y - 20 = 5(x - 1)$

$y = 5x + 15$

(ii) The table below represents points on another graph G_2.

Find the equation of this graph.

x	y
1	0
2	4
3	12
4	24
5	40

)4)8)12)16)4)4)4

2nd diff constant. Quadratic. $a = \frac{\text{2nd diff}}{2} = 2x^2$ while $2 = a$.

$c = 0$ so $y = 2x^2 + bx$, $0 = 2(1)^2 + b(1)$. $b = -2$

so $\therefore$ $y = 2x^2 - 2x$

(iii) **Use algebra**, to find the x-values of the two points of intersection of the graphs G_2 and G_1.

Support your answer with full mathematical working.

$y = 2x^2 - 2x$, $y = 5x + 15$. $2x^2 - 2x = 5x + 15$, $2x^2 - 7x - 15 = 0$. -30, -7 $-10\ 3$

$\frac{(2x - 10)(2x + 3)}{2} = 0$

$= (x - 5)(2x + 3) = 0$, $x = 5$, $x = -\frac{3}{2}$

(b) Using the set of axes provided below, draw the two graphs of $y = 3x^2 - 14x - 120$ and $y = 10x + 24$. (Plotted both with pencil)

Using your graphs, solve the equation $3x^2 - 14x - 120 = 10x + 24$.

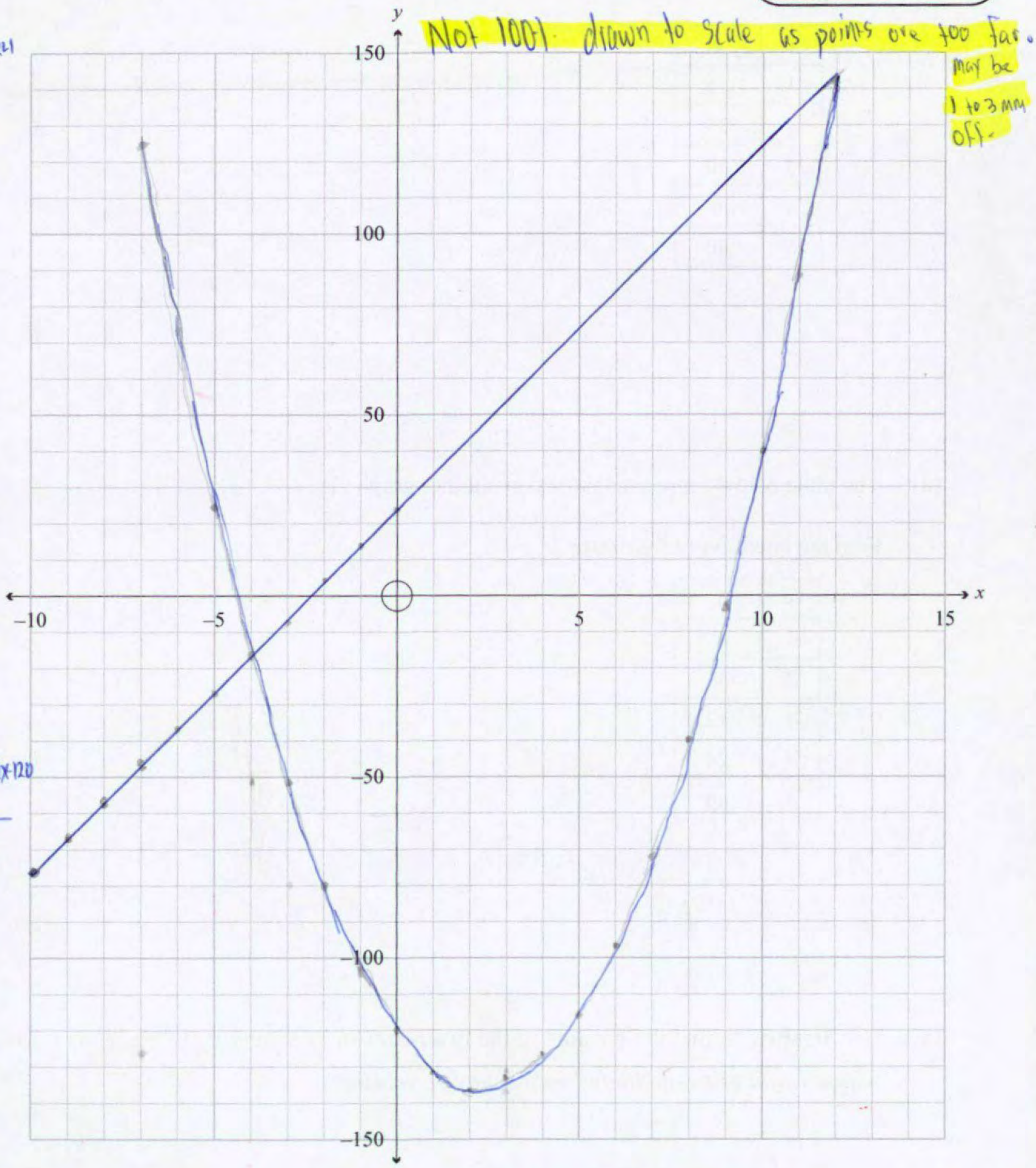
If you need to redraw your response, use the grid on page 12.

Table $10x+24$

X	Y
-10	-76
-9	-66
-8	-56
-7	-46
-6	-36
-5	-26
-4	-16
-3	-6
-2	4
-1	14
0	24
1	34
2	44
3	54
4	64
5	74
6	84
7	94
8	104
9	114
10	124
11	134
12	144
13	154
14	164
15	174

Table $y=3x^2-14x-120$

X	Y
-10	326
-9	249
-8	184
-7	125
-6	72
-5	25
-4	-16
-3	-51
-2	-80
-1	-103
0	-120
1	-131
2	-136
3	-135
4	-128
5	-115
6	-96
7	-71
8	-40
9	-3
10	40
11	89
12	144
13	205
14	272
15	345



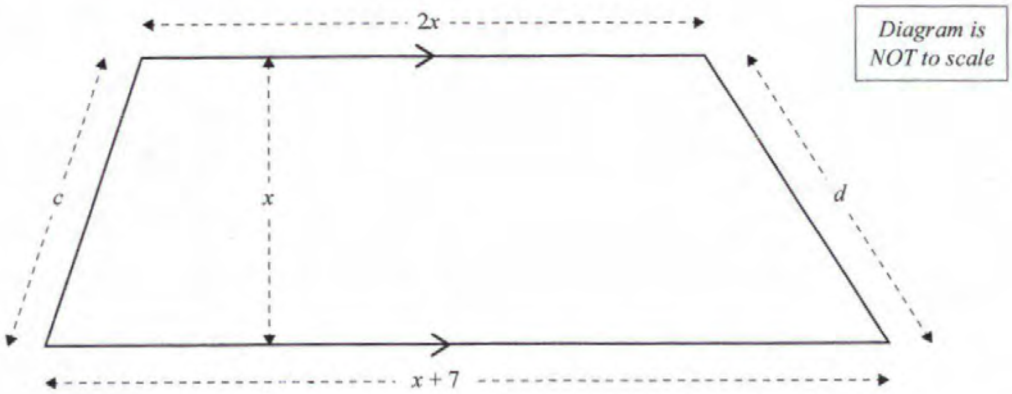
Visually seeing this, we can see that they intersect at $(-4,-16)$ and at $(12,144)$, graphs are just a visual way of solving equations, intersecting point = Solve so ∴ solving, $x=4, x=-12$.

Proof: $3x^2-14x-120=10x+24$, $3x^2-24x-144=0$ $(3x-36)(x+4)=0$, $x=12, x=4$.

Rule: Intersecting points = solving.

(c) The diagram below shows a trapezium with area of 20 m^2.

All lengths are in metres.



Find the value of x.

Support your answer with full mathematical working.

area trapezium $= \frac{1}{2}(a+b)h$.

$= \frac{1}{2}(2x + x + 7)x$

$= \frac{1}{2}(3x+7)(x)$

$= \frac{1}{2}(3x^2+7x) = \frac{3x^2+7x}{2} = 20$

So $3x^2+7x=40$, $3x^2+7x-240=0$.

Factor then solve

-120, 7

1 120
2 60
3 40
4 30
5 24
6 20
7 -
-8 $+15$

So $\underbrace{(3x+15)}_{3}(3x-8)=0$

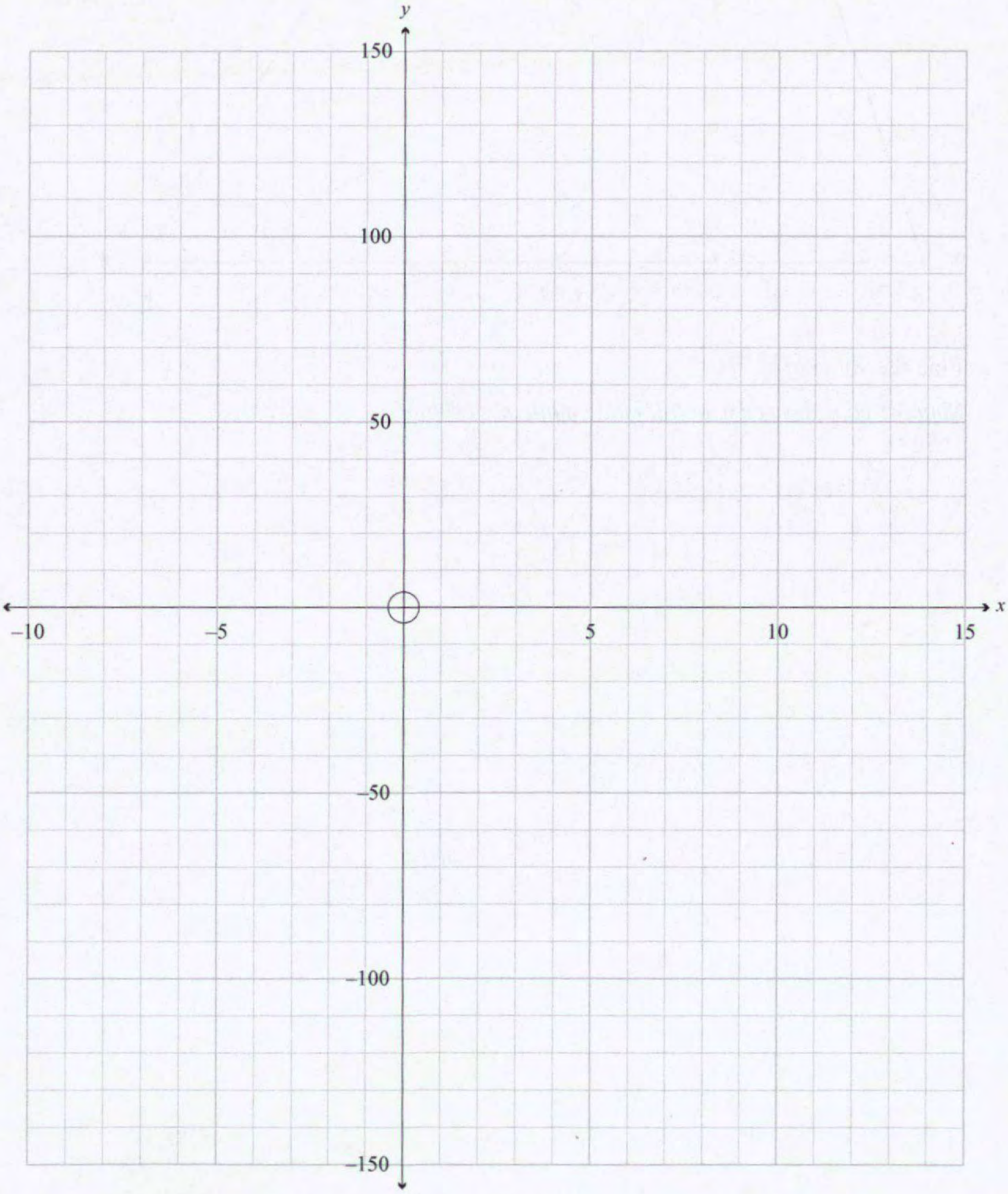
$= (x+5)(3x-8) = 0$.

$\boxed{x = \frac{8}{3}}$, $x = -5$.

$x \neq -5$. x must >0 due to dealing with potential real life situations $\therefore x = \frac{8}{3}$

SPARE DIAGRAM

If you need to redraw your response to Question Three (b), use the diagram below. Make sure it is clear which answer you want marked.



Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

Check end of page for working of 2bii.

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

Check last page for working of 2bii.

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

Check last page for working of 2bii.

$(50-x)(50-x) = 2500 - 50x - 50x + x^2$

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

Here is working.

2bii. (Use pythageros therom, find area of triangle ABC).

$A^2 + b^2 = C^2$, so $A = x$, $b = 10$, $C = y$.

so $x^2 + 10^2 = y^2$. $x^2 + 100 = y^2$. ~~$y = \sqrt{x^2 + 100}$~~

~~While perimeter $= 2x + 2y = 100$. $2x + 2(\sqrt{x^2+100})$~~

While perimeter $= 2x + 2y = 100$. $2y = 100 - 2x$. $y = \frac{100-2x}{2} = 50 - x$.

$x^2 + 10^2 = y^2$. so $x^2 + 100 = (50-x)^2 = x^2 + 100 = (x^2 - 100x + 2500)$

so $\cancel{x^2} + 100 = \cancel{x^2} - 100x + 2500$.
$-x^2 - 100 \quad -x^2 + 100x$
$+100x$

$100x = 2400$. $\underline{x = 24}$

Area $= \frac{1}{2}bh$.

$b = 2x$, $h = 10$. $\frac{2(24)(10)}{2} = 240cm^2$

91947

Excellence

Subject: Mathematics and Statistics RAS

Standard: 91947

Total score: 24

Q	Grade score	Marker commentary
One	E8	(a) Correct answer. (b)(i) Correct answer. (b)(ii) Provided correct volume of space. The candidate developed a chain of logical reasoning to calculate the volume of the box, NOT occupied by the tins in this box, with the radius of the tins given as a variable, thus forming a generalisation. (c)(i) correct answer with working. (c)(ii) correct bearing. (c)(iii) provided correct expression for SV. The candidate formed a generalisation to determine the speed of a ship, given the speed of another ship and the time taken for both ships to travel a given distance.
Two	E8	(a)(i) Clear and justified working to show that $v = 67.5°$. (a)(ii) Correct answer. (a)(iii) Finding a correct expression for the area of the whole polygon table. Candidate extended mathematical methods to solve the problem of providing a generalisation to determine the surface area of a polygon with n sides, given a length from the centre. Minor error ignored. (b)(i) Found y in terms of x. (b)(ii) The candidate used a chain of logical reasoning to correctly calculate the area of a triangle, extending mathematical methods to solve a problem.
Three	E8	(a)(i) Correct answer. (a)(ii) Correct answer. (a)(iii) Found both values of x. (b) Both intersection points identified accurately and with evidence of use of an accurate graph. The candidate extended mathematical methods (graphing) to solve an equation. (c) $x = \frac{8}{3}$ with evidence that $x = -5$ has been ignored. The candidate formed a generalisation to use the area of a trapezium and extended mathematical methods to solve a problem.