Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Level 1 Mathematics and Statistics RAS 2023

91947 Demonstrate mathematical reasoning

EXEMPLAR

Merit

TOTAL 15

QUESTION ONE

(a) Find the value of T in the formula $T = \pi\sqrt{\dfrac{h\sin x}{g}}$ when $h = 2.5$, $g = 9.81$, $x = 75°$, giving your answer correct to **four decimal places**.

$\pi\sqrt{\frac{2.5\sin 75°}{9.81}} = 4.8595$

$T = 4.8595$

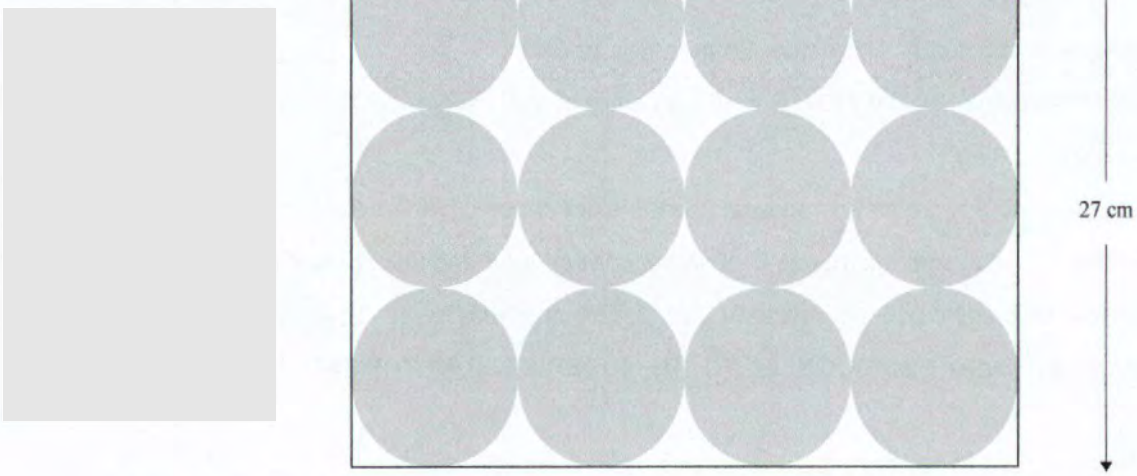
(b) The diagram below shows the top view of a rectangular box containing 12 cylindrical tins.
The tins are all just touching each other and the sides of the box. Each tin is 15 cm high.
Each tin has a label going all the way around its side, but not on the top or bottom.
The box has dimensions of 27 cm by 36 cm by 15 cm.

sphere $\frac{4}{3}\pi r^2$

cylinder $\pi r^2 h$

cone $\pi r^2 \frac{h}{3}$

Diagram is NOT to scale



Source: https://www.thewarehouse.co.nz/p/watties-condensed-tomato-soup-420g/R930548.html

height 15cm

cylinder surface area $= 2\pi rh + 2\pi r^2$

3

(i) Find the **total area** of the labels of all of the tins in the box.

side of cylinder surface area $= 2\pi rh$

~~2π~~ $36 \div 4 = 9$ $9/2 = 4.5$ $r = 4.5$cm

$2\pi \times 4.5 \times 15 = 424.12$ cm (2dp)

424.12×12 (cans) $= 5089.44\text{cm}^2$

(ii) A different size rectangular box to part (i) has height 15 cm.

The box will also contain 12 cylindrical tins, which are all just touching each other and the sides of the box. The layout of the 12 tins within this box will be the same as in part (i).

Each tin is 15 cm high, and with radius p cm.

Show that the **proportion** of the volume in the box that is NOT occupied by the tins is $\frac{(4-\pi)}{4}$.

$12\pi p^2 15$

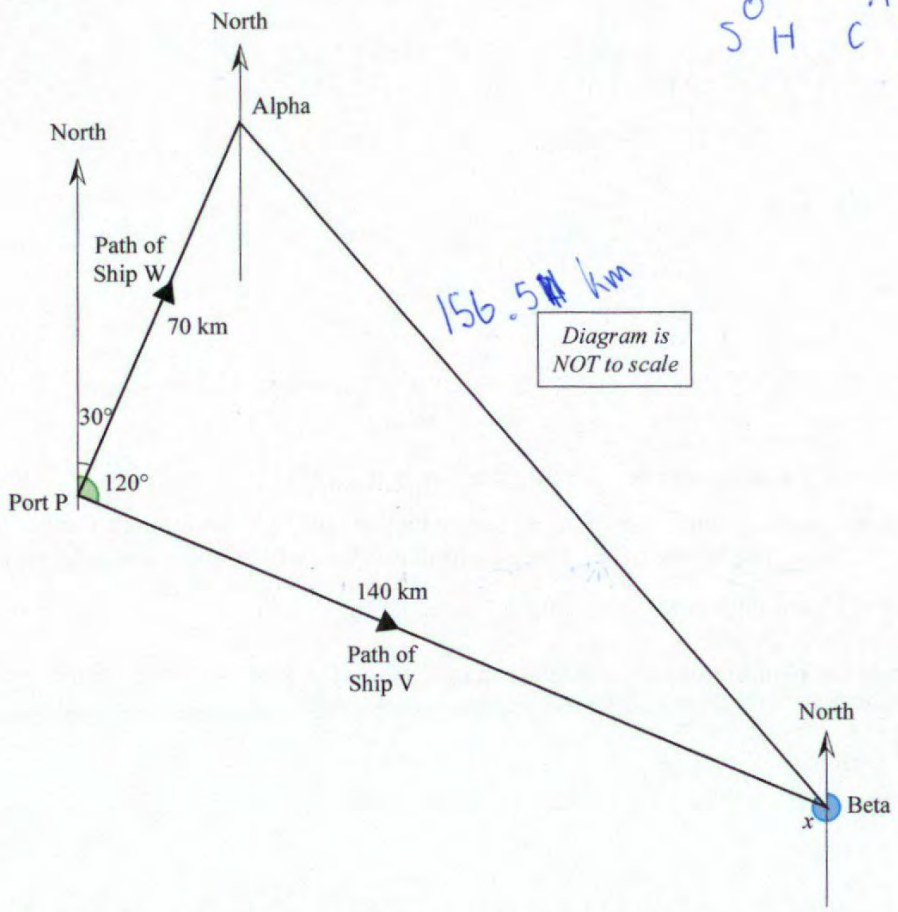
~~$12\pi p^2 h$~~

(c) Two ships leave Port P at the same time.

Ship W sails 70 km on a bearing of 030° to reach point Alpha.

Ship V sails 140 km on a bearing of 120° to reach point Beta.

SOH CAH TOA



156.5 km

4633

(i) Find the direct distance between the two places Alpha and Beta.

$a^2 + b^2 = c^2$

$70^2 + 140^2 = 24500$ $\sqrt{24500} = 156.5$ (1 dp)

Distance between Alpha and Beta = 156.5 km

(ii) Find the bearing of Alpha from Beta, shown as angle x in the diagram opposite.

Show your working clearly.

~~$\theta = \tan^{-1}\frac{\text{opp}}{\text{adj}}$~~ $\tan^{-1}\left(\frac{70}{140}\right) = 0.4634$ angle inside $x = 46.4°$

$46.4 - 360$ $\tan^{-1}\left(\frac{70}{140}\right) = 26.6$ (1dp) $26.6 - 360 = -333.4$

bearing $x = 333.4°$

(iii) The speed of ship W is k km/hour, where k is a positive constant.

The total time taken for the ships to complete their journeys to Alpha and Beta was four hours.

Find the speed of ship V, giving your answer in terms of k.

$k = \frac{d}{t}$ $\frac{70}{4} = 17.5$ km/h

$k = 17.5$

speed $= 17.5k$ km/h

QUESTION TWO

(a) The diagram below shows the top of a table which is in the shape of a regular octagon.
Length AZ = 120 cm. Point Z is at the centre of the octagon.

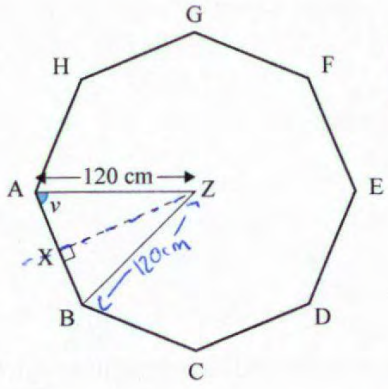


Diagram is NOT to scale

SOH CAH TOA

All to 1dp

(i) Show that the size of v, angle ZAB, is 67.5°.

Show your working clearly.

~~opposite~~ = ~~opposite = sin θ x hypotenuse~~ sin 67.5 X 120

~~Line BZ = 120cm~~ 360 ÷ 8 = 45 angle AZB = 45°

~~sin θ = $\frac{opp}{hyp}$~~ ~~sin^{-1}($\frac{120}{120}$)~~ 180 - 45 = 135

since it isoceles, 135 ÷ 2 = 67.5 angle ZAB = 67.5°

(ii) Find the area of the octagon.

finding half triangle: opp = hyp x sinθ 120 X sin67.5 = 110.9 (1dp)

line XZ = 110.9cm

$c^2 - b^2 = a^2$ $120^2 - 110.9^2 = 2101.2$ $\sqrt{2101.2} = 45.8$

line AX = 45.8cm

45.8 X 2 = 91.6cm line AB = 91.6cm

$\frac{1}{2} \times b \times h$ $\frac{1}{2} \times 91.6 \times 110.9 = 5079.2 cm^2$

$5079.2 \times 8 = 40633.6 cm^2$

(iii) Another table, made in the same style, has its top in the shape of an n-sided regular polygon.

The length AZ = p cm, where Z is at the centre of the table and A is one of the corners of the table.

$(n-2) \times 180^\circ$

Find the area of this new table top, giving your answer in terms of n and p.

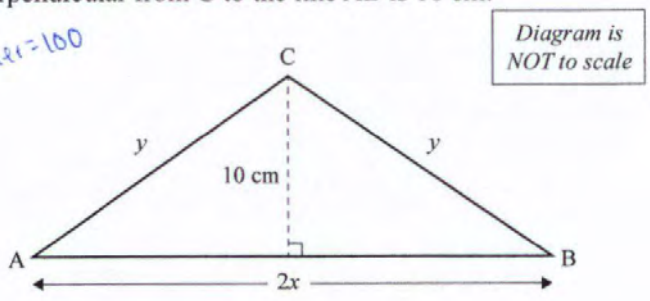
p=AZ p=120 p=AZ n=amount of sides

(b) An isosceles triangle ABC has AB = $2x$ cm and AC = BC = y cm.

The perimeter of the triangle ABC is 100 cm.

The length of the perpendicular from C to the line AB is 10 cm.

perimeter = 100



(i) Find the length, y, from A to C.

Give your answer in terms of x.

page 13

$x = y^2 - 10^2$ $\quad 2x + 2y = 100$ $\quad y + x = 50$ $\quad y + (50 - y) = 50$

$x^2 + 10^2 = y^2$ $\quad y + x = 50$ $\quad y + x = 50$ $\quad y^2 - 50y = 50$

$2x + 2y = 100$ $\quad 2x + 2(x + 50) = 100$ $\quad x = 50 - y$

$2x^2 - 2x + 50 = 100$

(ii) Using Pythagoras' theorem, find the area of the triangle ABC.

Support your answer with full mathematical working.

QUESTION THREE

(a) (i) The table below represents points on a particular graph, G_1.

Find the equation of this graph.

x	y
1	20
2	25
3	30
4	35
5	40

0 | 1 2 3 4 5
15 | 20 25 30 35 40
+5 +5 +5 +5 +5

$y = 5x + 15$

(ii) The table below represents points on another graph G_2.

Find the equation of this graph.

x	y
1	0
2	4
3	12
4	24
5	40

0 1 2 3 4 5
0 0 4 12 24 40
+4 +8 +12 +16
+4 +4 +4

$y = 2x^2 - 2x$

(iii) **Use algebra**, to find the x-values of the two points of intersection of the graphs G_2 and G_1.

Support your answer with full mathematical working.

$y = 5x + 15$
$y = 2x^2 - 2x$
$y = 2x + 2$
$y^2 =$

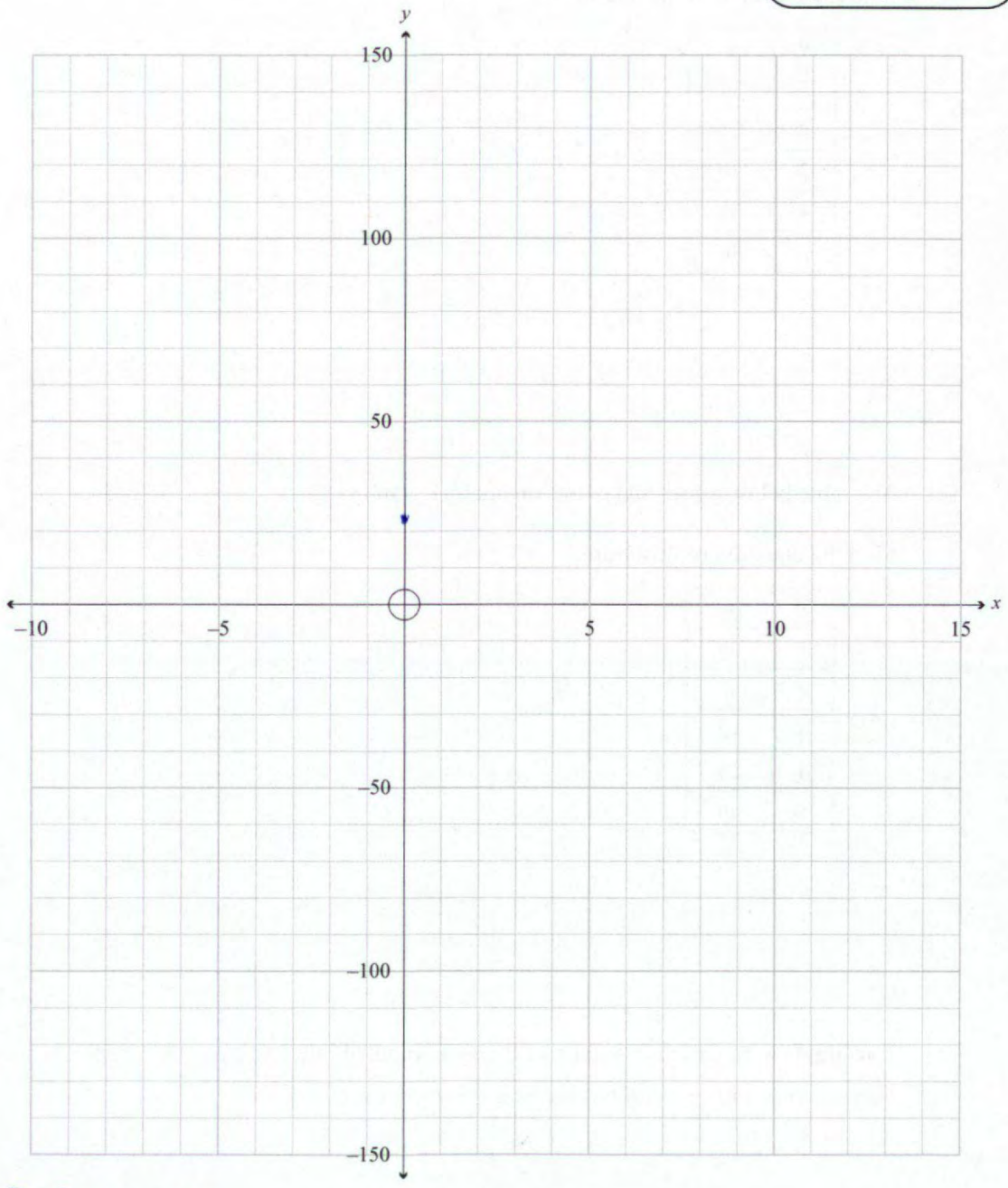
$y = 5x + 15$
$y = 2x - 2$

~~$y = 5x + 15$~~
~~$y = 2x + 2$~~ ~~$y = 2x - 2$~~
$2x - y = 2$
~~$y = 2x(5x + 15)$~~
$2x(5x + 15) = 2$

(b) Using the set of axes provided below, draw the two graphs of $y = 3x^2 - 14x - 120$ and $y = 10x + 24$.

Using your graphs, solve the equation $3x^2 - 14x - 120 = 10x + 24$.

If you need to redraw your response, use the grid on page 12.



360
123469

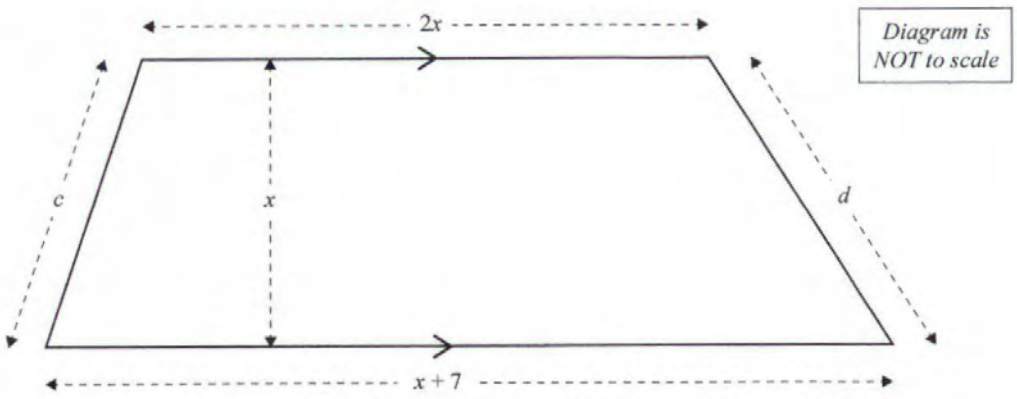
24
15

$3x^2 - 14x - 120$

$3x^2 -$

(c) The diagram below shows a trapezium with area of 20 m^2.

All lengths are in metres.



Find the value of x.

Support your answer with full mathematical working.

Merit

Subject: Mathematics and Statistics

Standard: 91947

Total score: 15

Q	Grade score	Marker commentary
One	M5	(a) Incorrect answer. (b)(i) Correct answer. (b)(ii) Found the volume of one tin or CAO. (c)(i) Correct answer with appropriate working. (c)(ii) Found, with appropriate working that ∠ABP = 26.57°. (c)(iii) Incorrect answer.
Two	M5	(a)(i) Clear and justified working to show that $v = 67.5°$. (a)(ii) Correct answer but with premature rounding. (a)(iii) Incorrect answer. (b)(i) Found correct equation involving x and y. (b)(ii) No response.
Three	M5	(a)(i) Correct answer. (a)(ii) Both values of x. (a)(iii) Incorrect answer. (b) Incorrect answer. (c) No response.